Trunking. Erlangs

- **Traffic engineering** uses statistical techniques such as queuing theory to predict and engineer the behaviour of telecommunications networks such as telephone networks or the Internet.

- **The consept of trunking** allows a large number of users to share relatively small number of channels in a cell by providing acces to each user. The fundamental of trunking theory were developped by Danish mathematician **A. K. Erlang.**

Example. A radio channel that is occupied fo4 30 minutes during 1 hour carries 30/60=0.5 Erlangs of trafic

- **The Grade of Service** is a measure of a ability of user to acces a trunked system during a bnusiest hour.

Example GOS=2% implies that 2 out of 100 call will be blocked due to channel occupancy.

Key Definitions for Trunked Radio

Definitions of Common Terms Used in Trunking Theory

Set-up Time: The time required to allocate a trunked radio channel to a requesting user.

Blocked Call: Call which cannot be completed at time of request, due to congestion. Also referred to as a *lost call*.

Holding Time: Average duration of a typical call. Denoted by H (in seconds).

Traffic Intensity: Measure of channel time utilization, which is the average channel occupancy measured in Erlangs. This is a dimensionless quantity and may be used to measure the time utilization of single or multiple channels. Denoted by A.

Load: Traffic intensity across the entire trunked radio system, measured in Erlangs.

Grade of Service (GOS): A measure of congestion which is specified as the probability of a call being blocked (for Erlang B), or the probability of a call being delayed beyond a certain amount of time (for Erlang C).

Request Rate: The average number of call requests per unit time. Denoted by λ seconds^{-1}.

Characterization of Telephone Traffic

The trafic intensity Au (Erlang) offered by each user:

$$\mathbf{A_u = \lambda H}$$

λ – Calling Rate or Arrival Rate- Average number of calls initiated per unit time

$$\lambda = \frac{N_{call}}{H}$$

– Number of calls in time T is **Poisson** distributed:

$$p(x) = \frac{e^{-\lambda}.\lambda^{x}}{x!} \qquad x=0,1,2,3,.....$$

Time between calls is exponential:

$$f(t) = \lambda.e^{-\lambda t} \qquad 0 \le t \le \infty \qquad mean = \frac{1}{\lambda}$$

•If receive N_{call} calls from **m** terminals in time H:

Group calling rate

$$\lambda_g = \frac{N_{call}}{H}$$

Per terminal calling rate

$$\lambda_g = \frac{N_{call}}{m.H}$$

For va system containing Nu users total offerd trafic intencity

$$\mathbf{A_{ol} = A_u N_u}$$

In a N_{Ch} channel trunked system $\mathbf{A}_{ol} = \mathbf{A_u}\,Nu\,/\mathbf{N}_{Ch}$

Erlangs

Dimensionless unit of traffic intensity

• Named after Danish mathematician **A. K. Erlang** (1878-1929)

Defined as one circuit occupied for one hour. 1 Erlang = 1 Call–hour / hour

Busy hour traffic

Erlangs = (Calls/busy hour)*(call holding time)

Example

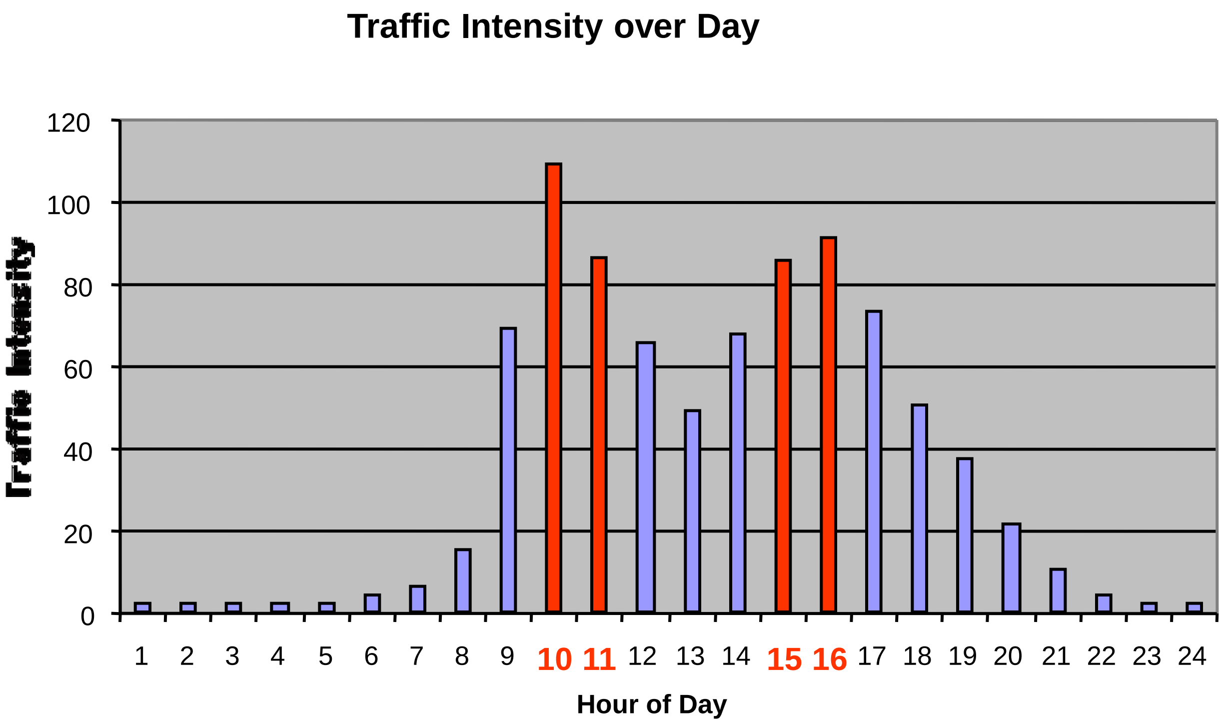
- A group of 20 subscribers generate 50 calls with an average holding time of 3 minutes, what is the average traffic per subscriber?
- Total Traffic = (50 calls)*(3min)/(1 hour)=50*3/60 = **2.5 Erlangs**
- = 2.5 / 20 or 0.125 Erlangs per subscriber.

Individual (residential) calling rates are quite low and may be expressed in milli-Erlangs, i.e. 0.125 Erlangs = 125 milli-Erlangs.

Busy Hour

- Busy hour is that continuous 60 minutes time span of the day during which the highest usage occurs.

(Dayly, weekly and seasonal variation)



There are 2 types of Trunked systems:

1. **Blocked call cleared sytem** if no channels available for requesting user the requesting user is blocked and is free to try again. To describe of this system Erlang B formula is used.

Erlang B Formula

- Simplest assumption that any blocked call is lost:

$$P_b = \frac{\frac{A^N}{N!}}{\sum_{i=0}^{N} \frac{A^i}{i!}}$$

where
A = Offered Traffic
N = Number of Servers (Lines)
P_b = Probability of Blocking

2. **Blocked call delayed sytem** - is one in which a que is provided to hold calls which are blocked. If a channel is not available immediately, the call request may be delayed until a channel becomes available.

Erlang B Sample Calculation

- A = 3 Erlangs
- N = 6 Lines
- P_b given by:

$$\frac{\frac{3^6}{6!}}{\frac{3^0}{0!}+\frac{3^1}{1!}+\frac{3^2}{2!}+\frac{3^3}{3!}+\frac{3^4}{4!}+\frac{3^5}{5!}+\frac{3^6}{6!}}$$

$$\frac{\frac{729}{720}}{1+3+\frac{9}{2}+\frac{27}{6}+\frac{81}{24}+\frac{243}{120}+\frac{729}{720}}$$

$$\frac{1.0125}{19.4125}=0.522 \quad \text{or} \quad \underline{5.22\%}$$

Note:
0! = 1
A^0 = 1

Erlang B Trunking GOS

Capacity of an Erlang B System

f	Capacity (Erlangs) for GOS			
Channels *C*	**= 0.01**	**= 0.005**	**= 0.002**	**= 0.001**
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

A for a given number of channels N_{ch}=1- 30 and blocking probability (GOS) of 0.5 % and 2%, trafic intencities are given below:

N	0.005	0.02	N	0. 005	0.02	N	0.005	0.02
1	.005	.021	11	4.62	5.84	21	11.9	14.0
2	.106	.224	12	5.28	6.62	22	12.6	14.9
3	.349	.603	13	5.96	7.41	23	13.4	15.8
4	.702	1.09	14	6.66	8.20	24	14.2	16.6
5	1.13	1.66	15	7.38	9.01	25	15.0	17.5
6	1.62	2.28	16	8.10	9.83	26	15.8	18.4
7	2.16	2.94	17	8.83	10.7	27	16.6	19.3
8	**2.73**	**3.63**	18	9.58	11.5	28	17.4	20.2
9	3.33	4.34	19	10.3	12.3	29	18.2	21.0
10	3.96	5.08	20	11.1	13.2	30	19.0	21.9

- A single GSM carrier supports 8 (TDM) speech channels.
- From the table on slide 9 we can see that for N=8 we can carry 3.63 Erlangs of traffic at 0.02 or 2.73 Erlangs at 0.005.
- How many 3 minutes calls does this represent?
  - GOS= 0.02, A_{cl}=3.63; A_{cl}= N_{call}* 3 / 60 or N_{call} = 72 calls
  - GOS=0.005, A_cl=2.73; A_{cl}= N_{call} * 3 / 60 or N_{call} = 54 calls

Erlang B

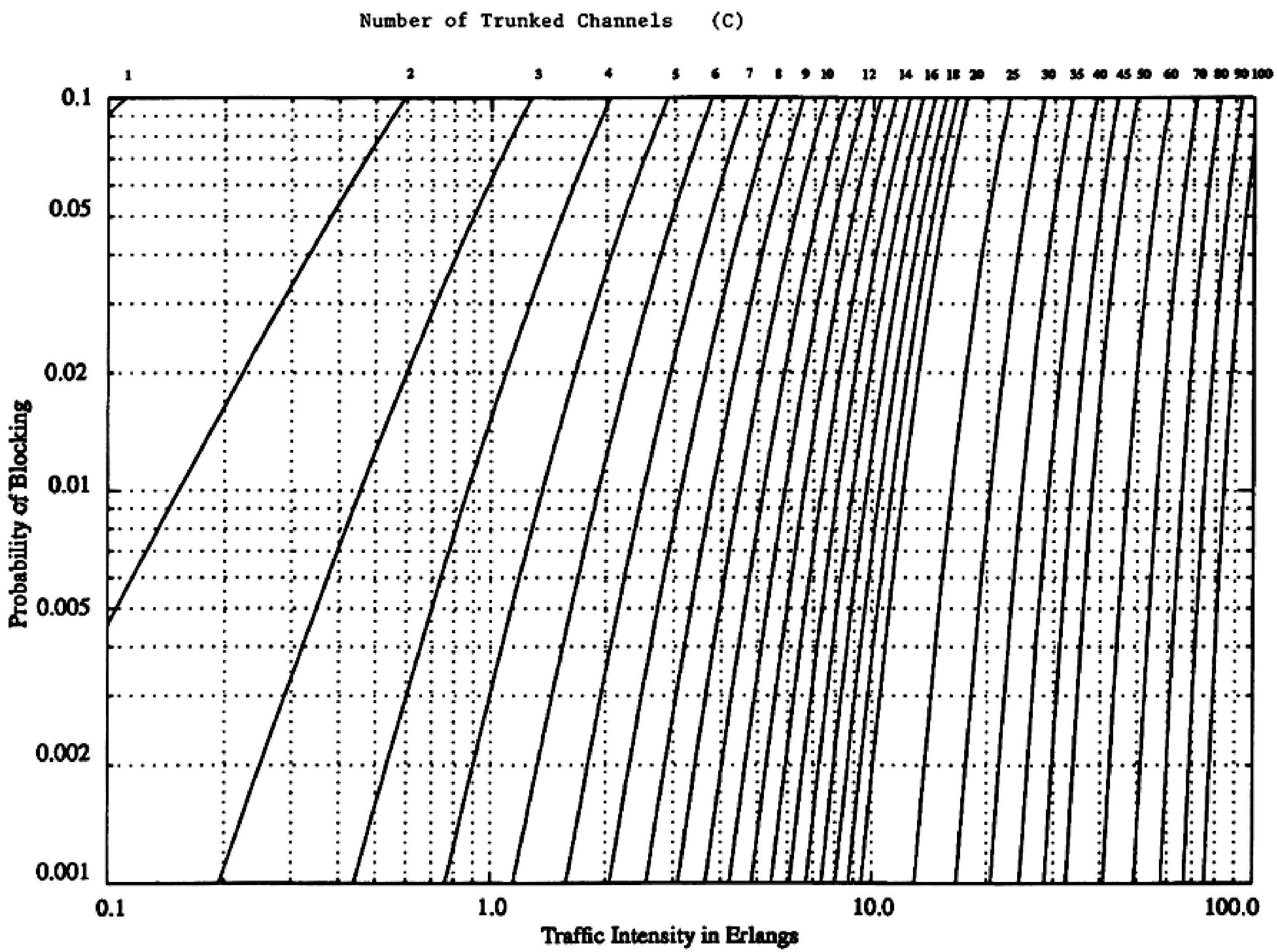


Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.

Erlang C Formula

The probability of a call not having immediately acces to a channel is defined by Erlang C formula

$$\mathbf{Pr}\left[\mathbf{delay} > \mathbf{0}\right] = \frac{\mathbf{A}^{\mathbf{C}}}{\mathbf{A}^{\mathbf{C}} + \mathbf{C}!\left(\mathbf{1} - \frac{\mathbf{A}}{\mathbf{C}}\right)\sum_{\mathbf{k}=\mathbf{0}}^{\mathbf{C}-\mathbf{1}} \frac{\mathbf{A}^{\mathbf{k}}}{\mathbf{k}!}}$$

The probability of a system where blocked call are delayed greater than t second:

Pr[delay>t] = Pr[delay>0] Pr[delay>t|delay>0]=

= **Pr[delay>0]exp(-(C-A)t/H)**

Theaverage call delay:

$$\mathbf{D} = \mathbf{P_r}\left[\mathbf{delay} > \mathbf{0}\right]\frac{\mathbf{H}}{\mathbf{C} - \mathbf{A}}$$

Erlang C

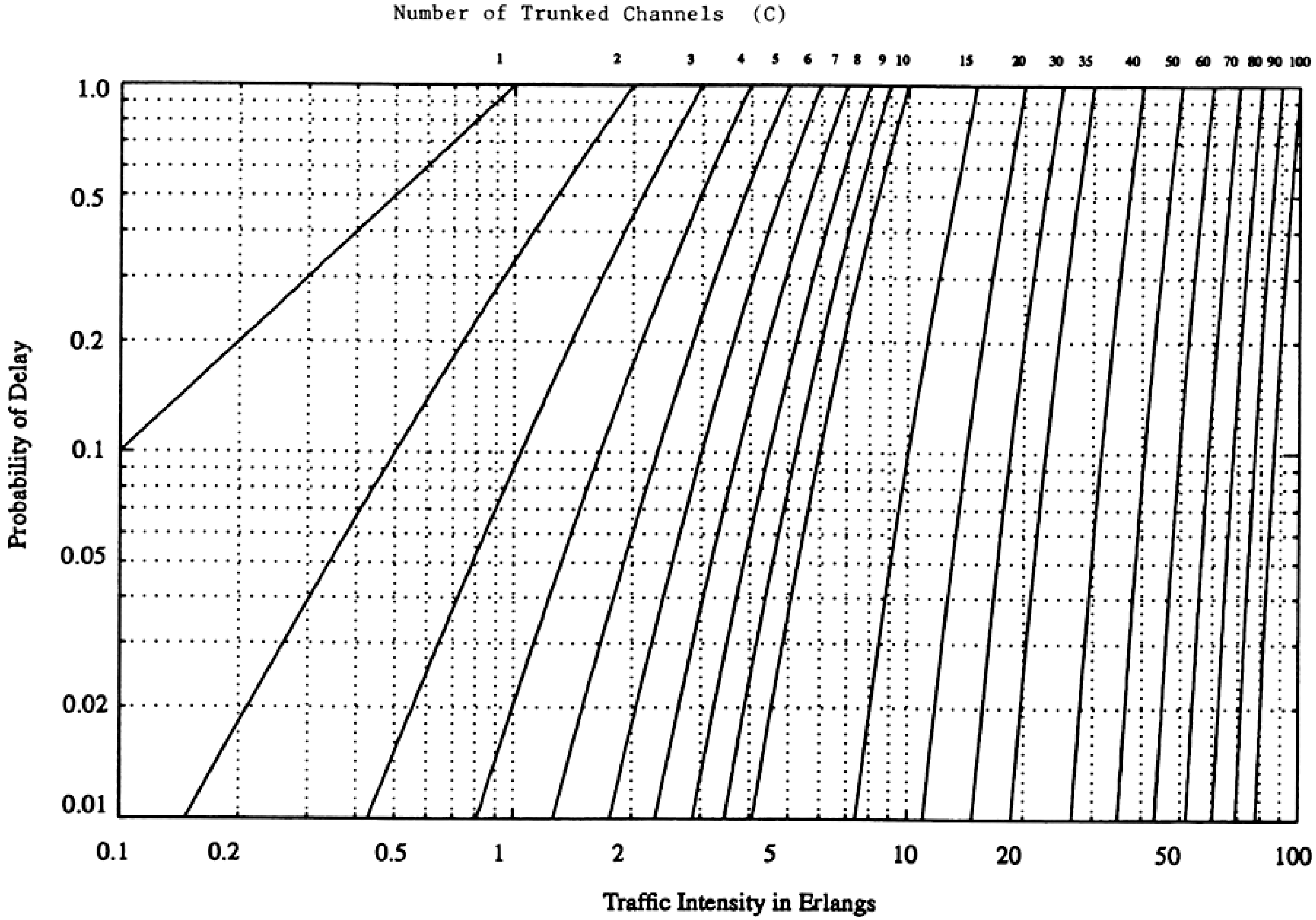


Figure 3.7 The Erlang C chart showing the probability of a call being delayed as a function of the number of channels and traffic intensity in Erlangs.